IIT JEE (ADVANCE) – 2017

PAPER-1

9

Time : 3 Hours **Maximum Marks : 183**

COVER PAGE IS AS PER THE ACTUAL PAPER

READ THE INSTRUCTIONS CAREFULLY

GENERAL

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at **9:00 am**, verify that the booklet contains **36** pages and that all the **54** questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon-less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
14. Write your name, roll number and code of the examination center, and sign with pen is the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

15. Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
16. Darken the bubble ◯ **COMPLETELY**.
17. The correct way of darkening a bubble is as: ●
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or "un-darken" a darkened bubble.

Please see the last page of this booklet for rest of the instruction.

PART I : PHYSICS

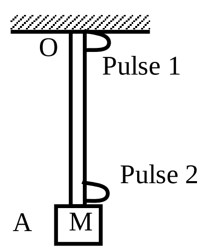
SECTION 1 (Maximum Marks: 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+ 4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+ 1	For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	– 2	In all other cases.

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get –2 marks, as a wrong option is also darkened.

Q.1 A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



[A] The time $T_{AO} = T_{OA}$
[B] The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope.
[C] The wavelength of Pulse 1 becomes longer when it reaches point A.
[D] The velocity of any pulse along the rope is independent of its frequency and wavelength.

Sol. **A, B, D**

Speed of transverse pulse at the point = $\sqrt{\dfrac{\text{Tension in rope at the point}}{\text{Linear mass density of rope}}}$

So, $T_{AO} = T_{OA}$
Wavelength becomes longer when speed of the pulse increases.

*Q.2 A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, and the value of σT_0^4 =460 Wm^{-2} (where σ is the Stefan-Boltzmann constant). Which of the following option is/are correct?

[A] The amount of energy radiated by the body in 1 second is close to 60 Joules.
[B] If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.

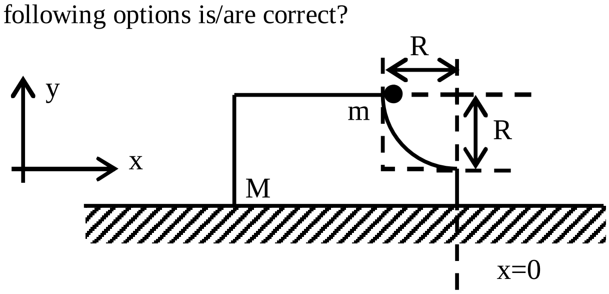
[C] Reducing the exposed surface area of the body (e.g by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation.

[D] If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths.

Sol. **C**

- Heat radiated by body remains unchanged even after change in room temperature.
- Energy lost by the radiation depends upon the surface area.
- From Wien's law $\lambda_m T$ = constant

*Q.3 A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



[A] The x component of displacement of the center of mass of the block M is : $-\frac{mR}{M+m}$.

[B] The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$.

[C] The velocity of the point mass m is : $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$.

[D] The velocity of the block M is: $V = -\frac{m}{M}\sqrt{2gR}$.

Sol. **A, C**

Δx_{cm} of the block & point mass system = 0

$\therefore \quad m(x+R) + Mx = 0$

where x is displacement of the block.

$x = -\frac{mR}{M+m}$

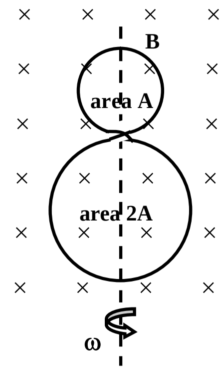
From conservation of momentum and mechanical energy of the combined system

$0 = mv - MV$

$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$

$\therefore \ v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$.

Q.4 A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



[A] The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper.
[B] The net emf induced due to both the loops is proportional to cos ωt.
[C] The emf induced in the loop is proportional to the sun of the areas of the two loops.
[D] The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone.

Sol. **A, D**

The net magnetic flux through the loops at time 't' is

$\phi = B(2A - A)\cos\omega t = BA\cos\omega t$

so, $\left|\frac{d\phi}{dt}\right| = B\omega A\sin\omega t$

$\therefore$ $\left|\frac{d\phi}{dt}\right|$ is maximum when $\phi = \omega t = \pi/2$

The emf induced in the smaller loop, $\varepsilon_{smaller} = -\frac{d}{dt}(BA\cos\omega) = B\omega A\sin\omega t$.

$\therefore$ Amplitude of maximum net emf induced in both the loops = Amplitude of maximum emf induced in the smaller loop alone.

Q.5 For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

[A] At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$.
[B] For this prism the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}(\mu/2)$.
[C] For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$.
[D] For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

Sol. **A, C, D**

The minimum deviation produced by a prism

$\delta_m = 2i - A = A$

$\therefore$ $i_1 = i_2 = A$ and $r_1 = r_2 = A/2$

$\therefore$ $r_1 = i_1/2$

Now using Snell's law

$\sin A = \mu\sin A/2$

$\Rightarrow \mu = 2\cos(A/2)$

For this prism when the emergent ray at the second surface is tangential to the surface

$i_2 = \pi/2$

$\therefore r_2 = \theta_c$

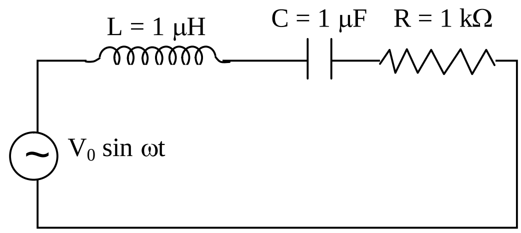
$\therefore r_1 = A - \theta_c$

so, $\sin i_1 = \mu \sin (A - \theta_c)$

so, $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism.

Q.6 In the circuit shown L = 1μH, C = 1μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



[A] The frequency at which the current will be in the phase with the voltage is independent of R.

[B] At ω~0 the current flowing through the circuit becomes nearly zero.

[C] At $\omega >> 10^6$ rad.s^{-1}, the circuit behave like a capacitor.

[D] The current will be in phase with the voltage if $\omega = 10^4$ rad. s^{-1}

Sol. **A, B**

$$I = \frac{v_0}{\sqrt{\left(L\omega - \frac{1}{C\omega}\right) + R^2}}$$

$$f_R = \frac{1}{\sqrt{LC}} \Rightarrow \text{Resonance frequency}$$

If $\omega \gg 10^6$ rad/s, then circuit behaves as inductive circuit.

*Q.7 A flat plate is moving normal to its plane through a gas under the action of constant force *F*. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true?

[A] The resistive force experienced by the plate is proportional to v

[B] The pressure difference between the leading and trailing faces of the plate is proportional to uv

[C] The plate will continue to move with constant non-zero acceleration, at all times

[D] At a later time the external force *F* balances the resistive force.

Sol. **A, B, D**

SECTION 2 (Maximum Marks : 15)

- This section contains **FIVE** questions
- The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive

- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened

Zero Marks : 0 In all other cases

*Q.8 A drop of liquid of radius R = 10^{-2} m having surface tension $S = \frac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is

Sol. **6**

$\Delta U = U_f - U_i$

$= S \times 4\pi R^2 \left[K^{1/3} - 1\right]$

$\Rightarrow K^{1/3} = \frac{\Delta U}{4\pi R^2 S} + 1 = 101$

$\Rightarrow K \approx 10^6$

Q.9 ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4 × 10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| \ll 1$ and ln 2 ≈ 0.7).

Sol. **5**

Final activity,

$$A_f = \frac{v}{v_{body}} \times A_0 \times e^{-\lambda t}$$

$$\Rightarrow v_{body} = \frac{v}{A_f} \times A_0 e^{-\frac{\ell n(2) \times t}{192}}$$

$= 4.998 \approx 5$ litres

Q.10 An electron in a hydrogen atom undergoes a transition from an orbit with quantum numebr n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electon. If $\frac{v_i}{v_f} = 6.25$, then the *smallest possible n_f* is

Sol. **5**

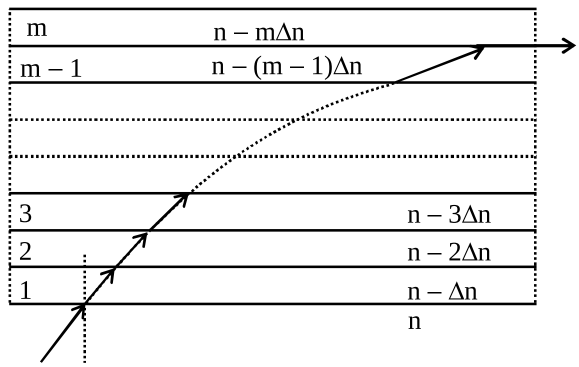
$\frac{v_i}{v_f} = 6.25$

$\Rightarrow \left(\frac{n_f}{n_i}\right)^2 = 6.25$

$\Rightarrow \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$

$\Rightarrow$ minimum value of n_f is 5.

Q.11 A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle θ = 30°. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m?



Sol. **8**

$n \sin\theta = (n - m\Delta n) \times \sin 90°$

$\Rightarrow \quad m = \dfrac{n}{2\Delta n}$

$\Rightarrow \quad m = 8$

*Q.12 A stationary source emits sound of frequency f_0 = 492 Hz. The sound is *reflected* by a large car *approaching* the source with a speed of 2 ms^{-1}. The reflected signal is received by the soruce and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency *it* has received).

Sol. **6**

Frequencey received by approaching car

$$f_1 = f_0\left[1+\frac{2}{330}\right]$$

Frequency received by source again

$$f_2 = \frac{f_1}{\left(1-\frac{2}{330}\right)}$$

So, beat frequency $f_B = f_2 - f_0 = 6$ Hz.

SECTION 3 (Maximum Marks : 18)

- This section contains **SIX** questions of matching type
- The section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following catogories:

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened
Zero Marks : 0 If none of the bubbles is darkened
Negative Marks : –1 In all other cases

Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.		
Column I	**Column 2**	**Column 3**
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0^2\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

Q.13 In which case will the particle move in a straight line with constant velocity?

[A] (II) (iii) (S)
[B] (IV) (i) (S)
[C] (III) (ii) (R)
[D] (III) (iii) (P)

Sol. **A**

For constant velocity

$$q\left[\vec{E} + \left(\vec{V} \times \vec{B}\right)\right] = 0$$

Q.14 In which case will the particle describe a helical path with axis along the positive z direction?

[A] (II) (ii) (R)
[B] (IV) (ii) (R)
[C] (IV) (i) (S)
[D] (III) (iii) (P)

Sol. **C**

For helical path, having its axis along z-axis, B must be along z-axis.

Q.15 In which case would be particle move in a straight line along the negative direction of y-axis (i.e., more along $-\hat{y}$) ?

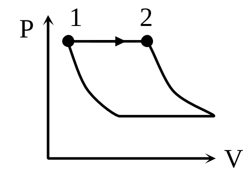
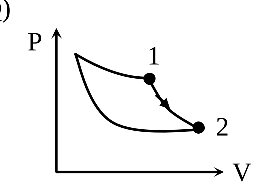
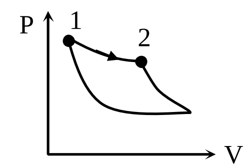
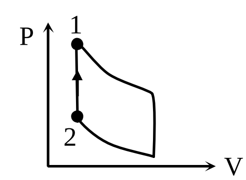
[A] (IV) (ii) (S)
[B] (III) (ii) (P)
[C] (II) (iii) (Q)
[D] (III) (ii) (R)

Sol. **D**

$\vec{E}$ and $\vec{B}$ must be along same line (i.e. parallel or anti parallel) with initial velocity either zero or along the field ($\vec{E}$ as well $\vec{B}$).

Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding *P* – *V* diagrams in column 3 of the table. Consider only the path from state 1 to 2. *W* denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is *n*.

Column I	Column 2	Column 3
(I) $W_{1-2} = \frac{1}{\gamma - 1}\left(P_2V_2 - P_1V_2\right)$	(i) Isothermal	(P) 
(II) $W_{1\to 2} = -PV_2 + PV_1$	(ii) isochoric	(Q) 
(III) $W_{1\to 2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to 2} = -nRT\ln(\frac{V_2}{V_1})$	(iv) Adiabatic	(S) 

*Q.16 Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

[A] (II) (iv) (R) [B] (II) (iii) (P)

[C] (II) (iii) (S) [D] (III) (iii) (P)

Sol. **B**

The process must be isobaric.

*Q.17 Which one of the following options is the correct combination?

[A] (III) (ii) (S) [B] (II) (iv) (R)

[C] (II) (iv) (P) [D] (IV) (ii) (S)

Sol. **A**

The correct combination is for isochoric process.

*Q.18 Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

[A] (III) (iv) (R) [B] (I) (ii) (Q)

[C] (IV) (ii) (R) [D] (I) (iv) (Q)

Sol. **D**

The process must be adiabatic, which is used in Laplace correction over Newton's law for sound speed in air.

PART II: CHEMISTRY

SECTION 1 [Maximum Marks: 28]

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

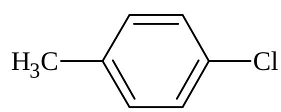
  Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened

  Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened

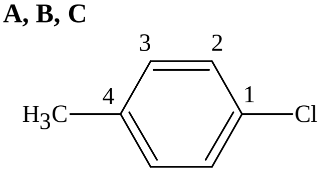
  Negative Marks : –2 In all other cases
- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will result in +4 marks; darkening only [A] and [D] will result in +2 marks; and darkening [A] and [B] will result in –2 marks, as a wrong option is also darkened.

*Q.19 The IUPAC name(s) of the following compound is(are)

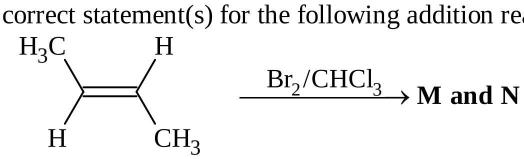


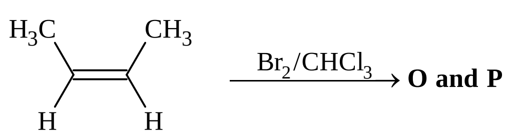
[A] 4-methylchlorobenzene [B] 4-chlorotoluene
[C] 1-chloro-4-methylbenzene [D] 1-methyl-4-chlorobenzene

Sol. **A, B, C**



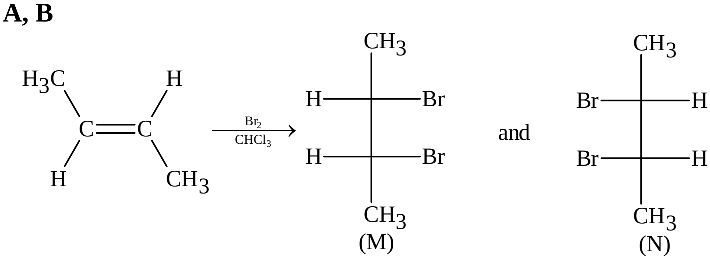
*Q.20 The correct statement(s) for the following addition reactions is(are)

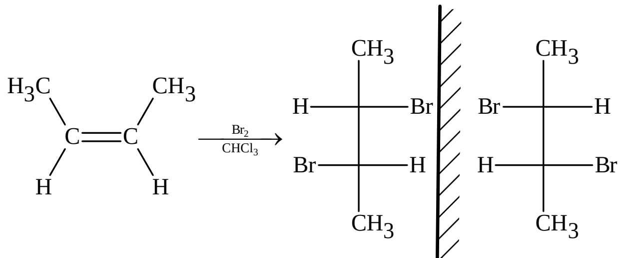
(i)


(ii)


[A] (**M** and **O**) and (**N** and **P**) are two pairs of diastereomers
[B] Bromination proceeds through *trans*-addition in both the reactions
[C] **O** and **P** are identical molecules
[D] (**M** and **O**) and (**N** and **P**) are two pairs of enantiomers

Sol. **A, B**





(O) (P)

(M and O) and (N and P) have no mirror image relationship. Hence these two pairs are diastereomers. Bromination proceeds through trans-addition in both the reactions.

Q.21 Addition of excess aqueous ammonia to a pink coloured aqueous solution of $MCl_2.6H_2O$(**X**) and NH_4Cl gives an octahedral complex **Y** in the presence of air. In aqueous solution, complex **Y** behaves as 1 :3 electrolyte. The reaction of **X** with excess HCl at room temperature results in the formation of a blue coloured complex **Z**. The calculated spin only magnetic moment of **X** and **Z** is 3.87 B.M., whereas it is zero for complex **Y**.
Among the following options, which statement(s) is(are) correct?

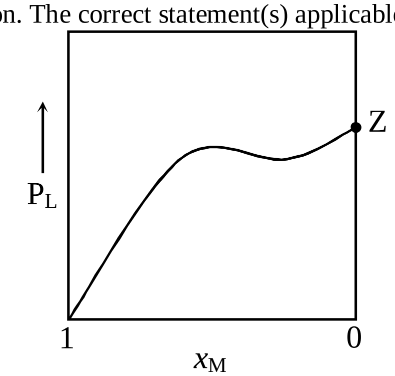
[A] The hybridization of the central metal ion in **Y** is d^2sp^3
[B] **Z** is a tetrahedral complex
[C] Addition of silver nitrate to **Y** gives only two equivalents of silver chloride
[D] When **X** and **Z** are in equilibrium at 0^0C, the colour of the solution is pink

Sol. **A, B, D**

$$\underset{\substack{(X)\\ \text{Octahedral(pink)}\\ (\mu=3.87\text{ B.M.})}}{\left[Co(H_2O)_6\right]Cl_2} \xrightarrow[\text{air}]{\text{Aq.}NH_3+NH_4Cl} \underset{\substack{(Y)\\ (\mu=0)}}{\left[Co(NH_3)_6\right]Cl_3}$$

$$\underset{(X)}{\left[Co(H_2O)_6\right]^{2+}} + HCl(\text{excess}) \xrightleftharpoons{0^o C} \underset{\substack{\text{tetrahedral(blue)}\\ (\mu=3.87\text{ B.M.})}}{\left[CoCl_4\right]^{2-}}, \ \Delta H = +ve$$

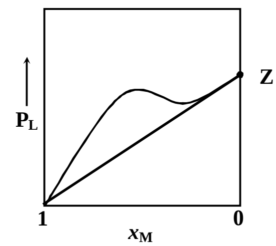
Q.22 For a solution formed by mixing liquids **L** and **M,** the vapour pressure of **L** plotted against the mole fraction of **M** in solution is shown in the following figure. Here x_L and x_M represent mole fractions of **L** and **M**, respectively, in the solution. The correct statement(s) applicable to this system is(are)



[A] Attractive intermolecular interactions between **L-L** in pure liquid **L** and **M-M** in pure liquid **M** are stronger than those between **L-M** when mixed in solution
[B] The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed when $x_L \rightarrow 0$
[C] The point **Z** represents vapour pressure of pure liquid **L** and Raoult's law is obeyed when $x_L \rightarrow 1$
[D] The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed from $x_L = 0$ to $x_L = 1$

Sol. **A, C**

From graph it is clear that there is +ve deviation w.r.t L. Therefore option A is correct. When $x_L \rightarrow 1$, then Z will have value equal to P_L^0 (vapour pressure of pure L). Therefore option C is also correct.



*Q.23 An ideal gas is expanded from (p_1, V_1, T_1) to (p_2, V_2, T_2) under different conditions. The correct statement(s) among the following is(are)

[A] The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1

[B] The work done by the gas is less when it is expanded reversibly from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversibly from V_1 to V_2 under isothermal conditions

[C] The change in internal energy of the gas is (i) zero, if it is expanded reversibly with $T_1 = T_2$, and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$

[D] If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic

Sol. **A, B, D**

Q.24 The correct statement(s) about the oxoacids, $HClO_4$ and HClO, is(are)

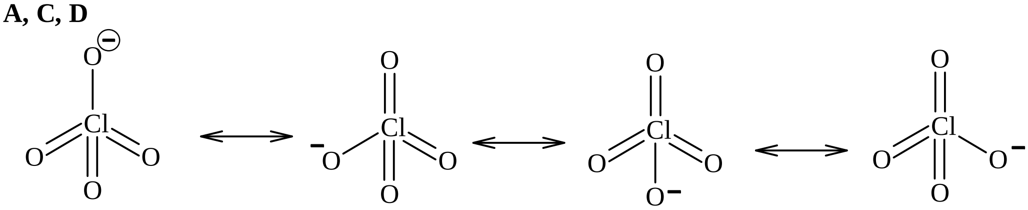
[A] $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

[B] $HClO_4$ is formed in the reaction between Cl_2 and H_2O

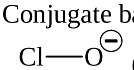
[C] The central atom in both $HClO_4$ and HClO is sp^3 hybridized

[D] The conjugate base of $HClO_4$ is weaker base than H_2O

Sol. **A, C, D**



Conjugate base of $HClO_4$ has four canonical structures.

 (Conjugate base of HOCl) is not resonance stabilized

$\Rightarrow$ The central atoms Cl in $HClO_4$ and O in HOCl respectively are sp^3 hybridized.

$\Rightarrow$ $HClO_4$ is stronger acid than H_3O^+, so ClO_4^- is weaker base than H_2O.

Q.25 The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to

[A] the physical state of X_2 at atom temperature changes from gas to solid down the group

[B] decrease in HOMO-LUMO gap down the group

[C] decrease in π^*-σ^* gap down the group

[D] decrease in ionization energy down the group

Sol. **B, C**

Highest occupied molecular orbital (HOMO) $\Rightarrow \pi^*$

Lowest unoccupied molecular orbital (LUMO) $\Rightarrow \sigma^*$

On descending the group gap between π^* and σ^* decreases.

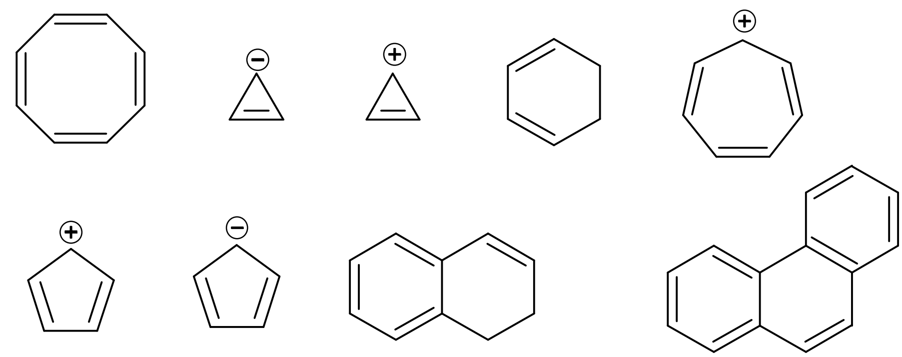
SECTION – 2 [Maximum Marks: 15]

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 In all other cases.

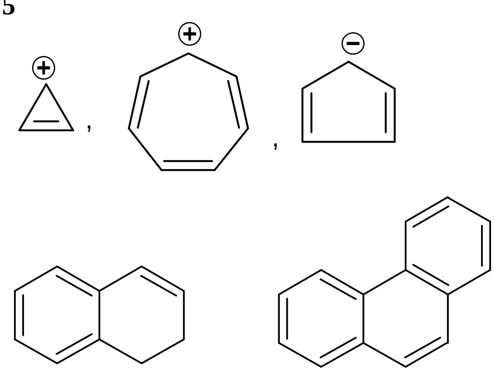
*Q.26 Among $H_2, He_2^+, Li_2, Be_2, B_2, C_2, N_2, O_2^-$, and F_2, the number of diamagnetic species is
(Atomic numbers: H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, F = 9)

Sol. **6**
H_2, Li_2, Be_2, C_2, N_2 and F_2 are diamagnetic species.
* However because Be_2 does not exits the answer may well be 5

*Q.27 Among the following, the number of aromatic compound(s) is



Sol. **5**



Q.28 The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be 5×10^{-7} S. The pH of the solution is 4. The value of limiting molar conductivity $\left(\Lambda_m^o\right)$ of this weak monobasic acid in aqueous solution is $Z \times 10^2$ S cm^{-1} mol^{-1}. The value of Z is

Sol. **6**

$$\kappa = G \times \frac{\ell}{a}$$

$$\kappa = 5 \times 10^{-7} \times \frac{120 \text{ cm}}{1 \text{ cm}^2} = 6 \times 10^{-5} \text{ S cm}^{-1}$$

$$\Lambda_m^C = \frac{\kappa \times 1000}{C} = \frac{6\times10^{-5}\times1000}{0.0015}$$

pH = 4, $[H^+] = 10^{-4} = C\alpha$

$$\alpha = \frac{10^{-4}}{0.0015}$$

$$\alpha = \frac{\Lambda_m^C}{\Lambda_m^o}$$

$$\frac{10^{-4}}{0.0015} = \frac{6\times10^{-5}\times1000}{0.0015\times\Lambda_m^o}$$

$\Lambda_m^0 = 6 \times 10^2$

Z = 6

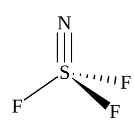
Q.29 The sum of the number of lone pairs of electrons on each central atom in the following species is

$$[TeBr_6]^{2-}, [BrF_2]^{+}, SNF_3, \text{ and } [XeF_3]^{-}$$

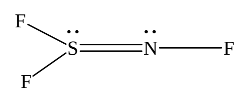
(Atomic numbers : N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54)

Sol. **6**

Species		Number of lone pairs
$[TeBr_6]^{-2}$	→	1
$[BrF_2]^{+1}$	→	2
SNF_3	→	0



most stable



however an alternate structure also exits

$[XeF_3]^{-1}$	→	3

$\therefore$ Sum is = 1 + 2 + 0 + 3 = 6 lone pair

Q.30 A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 400 pm. If the density of the substance in the crystal is 8 g cm^{-3}, then the number of atoms present in 256 g of the crystal is $N \times 10^{24}$. The value of N is

Sol. **2**

$$d = \frac{Z\times M}{N_A \times a^3}$$

$$8 = \frac{4\times M}{6.022\times10^{23}\times(400\times10^{-10})^3}$$

M = 76.8 g mol^{-1}

76.8 g contain = 6×10^{23} atoms

$\therefore$ 256 g will contain = 20×10^{23} atoms

= 2×10^{24} atoms

$\therefore$ N = 2

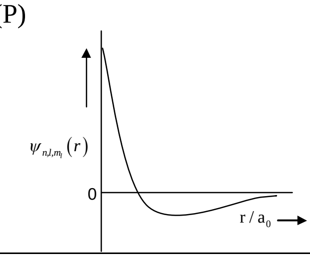
SECTION 3 (Maximum Marks: 18)

- This section contains **SIX** questions
- The section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened
Zero Marks : 0 If none of the bubbles is darkened
Negative Marks : - 1 In all other cases

Answer Q. 31, Q. 32 and Q. 33 by appropriately matching the information given in the three columns of the following table.

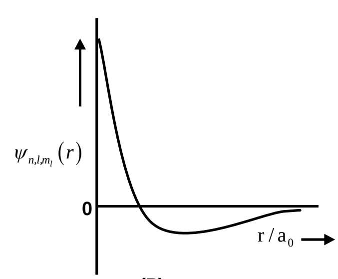
The wave function ψ_{n,l,m_l} is a mathematical function whose value depends upon spherical polar coordinates (r, θ, ϕ) of the electron and characterized by the quantum numbers n, l and m_l. Here r is distance from nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the Table, Z is atomic number and a_0 is Bohr radius.

Column 1	Column 2	Column 3
(I) 1s orbital	(i) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$	(P) 
(II) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_0^3}$
(III) $2p_z$ orbital	(iii) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{5}{2}} re^{-\left(\frac{Zr}{2a_0}\right)} \cos\theta$	(R) Probability density is maximum at nucleus
(IV) $3d_z^2$ orbital	(iv) *xy*-plane is a nodal plane	(S) Energy needed to excite electron from n = 2 state to n = 4 state is $\frac{27}{32}$ times the energy needed to excite electron from n = 2 state to n = 6 state

*Q.31 For the given orbital in Column 1, the only CORRECT combination for any hydrogen-like species is
[A] (IV) (iv) (R) [B] (II) (ii) (P) [C] (III) (iii) (P) [D] (I) (ii) (S)

Sol. **B**

$\underset{(II)}{\text{2s orbital}} - \underset{(ii)}{\text{One radial node}}\left(n-\ell-1\right)$



(P)

*Q.32 For He^+ ion, the only **INCORRECT** combination is

[A] (II) (ii) (Q) [B] (I) (i) (S) [C] (I) (i) (R) [D] (I) (iii) (R)

Sol. **D**

1s orbital can not have θ function (angular function).
Therefore D is incorrect.

*Q.33 For hydrogen atom, the only CORRECT combination is

[A] (I) (iv) (R) [B] (I) (i) (P) [C] (II) (i) (Q) [D] (I) (i) (S)

Sol. **D**

For H-atom:

$$\underset{(I)}{\text{1s orbital}} - \Psi_{n\ell m} \propto \underset{(i)}{\left(\frac{Z}{a_0}\right)^{3/2}} e^{-\left(\frac{Zr}{a_0}\right)}, S$$

$$E_4 - E_2 = -\frac{13.6}{16} - \left(-\frac{13.6}{4}\right) = \frac{3\times 13.6}{16}$$

$$E_6 - E_2 = -\frac{13.6}{36} - \left(-\frac{13.6}{4}\right) = \frac{8\times 13.6}{36}$$

$E_4 - E_2$ is $\frac{27}{32}$ times of $E_6 - E_2$

Answer Q. 34, Q. 35 and Q. 36 by appropriately matching the information given in the three columns of the following table.

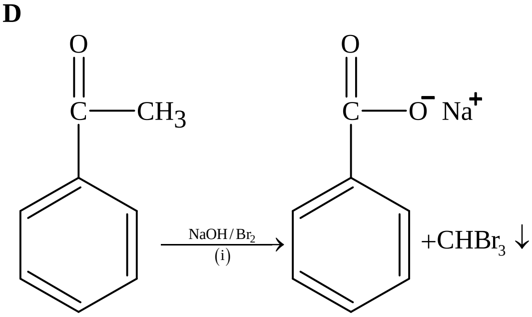
Columns 1, 2 and 3 contain starting materials, reaction conditions, and type of reactions, respectively.

Column 1	Column 2	Column 3
(I) Toluene	(i) $NaOH/Br_2$	(P) Condensation
(II) Acetophenone	(ii) $Br_2/h\nu$	(Q) Carboxylation
(III) Benzaldehyde	(iii) $(CH_3CO)_2O/CH_3COOK$	(R) Substitution
(IV) Phenol	(iv) $NaOH/CO_2$	(S) Haloform

Q.34 For the synthesis of benzoic acid, the only CORRECT combination is

[A] (III) (iv) (R) [B] (IV)(ii) (P) [C] (I) (iv) (Q) [D] (II) (i) (S)

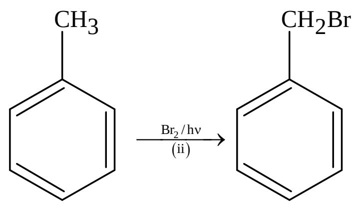
Sol. **D**



(II) Haloform reaction (S)

Q.35 The only CORRECT combination in which the reaction proceeds through radical mechanism is
[A] (I) (ii) (R) [B] (II) (iii) (R) [C] (III) (ii) (P) [D] (IV) (i) (Q)

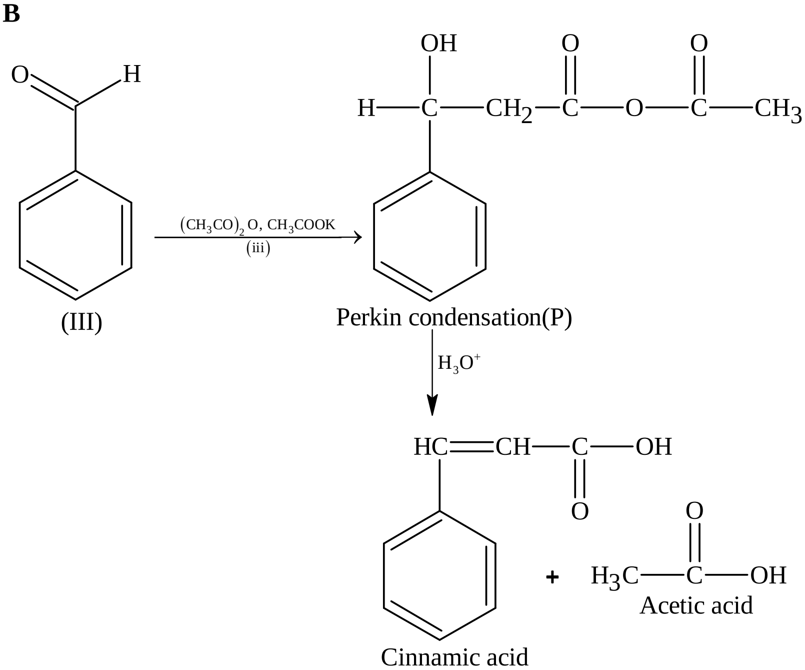
Sol. **A**



(I) Free radical substitution(R)

Q.36 The only CORRECT combination that gives two different carboxylic acids is
[A] (IV) (iii) (Q) [B] (III) (iii) (P) [C] (II) (iv) (R) [D] (I) (i) (S)

Sol. **B**



PART III : MATHEMATICS

SECTION 1 (Maximum Marks: 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+ 4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+ 1	For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	– 2	In all other cases.

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get –2 marks, as a wrong option is also darkened.

Q.37 Which of the following is(are) NOT the square of a 3 × 3 matrix with real entries ?

[A] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ [B] $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

[C] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ [D] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Sol. **A, B**

$|A|^2$ cannot be –ve

⇒ A and B option cannot be square of matrix

C option is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}^2$

D option is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix}^2$

* Q.38 If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?

[A] $p = 5, h = 4, k = -3$ [B] $p = -1, h = 1, k = -3$

[C] $p = -2, h = 2, k = -4$ [D] $p = 2, h = 3, k = -4$

Sol. **D**

$y^2 = 16x$

$2x + y = p$ … (i)

Equation of chord having mid point (h, k) is $T = S_1$

$yk - 8(x + h) = k^2 - 16h$

⇒ $yk - 8x = k^2 - 8h$ … (ii)

Comparing (i) and (ii)

$$\frac{k}{1} = -\frac{8}{2} = \frac{k^2 - 8h}{p}$$

⇒ $k = -4$ and $k^2 - 8h = -4p$

⇒ $16 - 8h = -4p$

⇒ $2h - p = 4$

Clearly h = 3, p = 2.

* Q.39 Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

[A] $-1-\sqrt{1-y^2}$ [B] $1+\sqrt{1+y^2}$

[C] $1-\sqrt{1+y^2}$ [D] $-1+\sqrt{1-y^2}$

Sol. **A, D**

$\text{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$

⇒ $(x + 1)^2 + y^2 = 1$

⇒ $x = -1 \pm \sqrt{1-y^2}$

Q.40 Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

[A] $P(X'|Y) = \frac{1}{2}$ [B] $P(X \cap Y) = \frac{1}{5}$

[C] $P(X \cup Y) = \frac{2}{5}$ [D] $P(Y) = \frac{4}{15}$

Sol. **A, D**

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$$

$$P\left(\frac{Y}{X}\right) = \frac{P(X \cap Y)}{P(X)} = \frac{2}{5}$$

$\Rightarrow P(X \cap Y) = \frac{2}{15}$

$P(Y) = \frac{4}{15}$ also $P(X \cup Y) = \frac{7}{15}$

$$P\left(\frac{X'}{Y}\right) = 1 - P\left(\frac{X}{Y}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

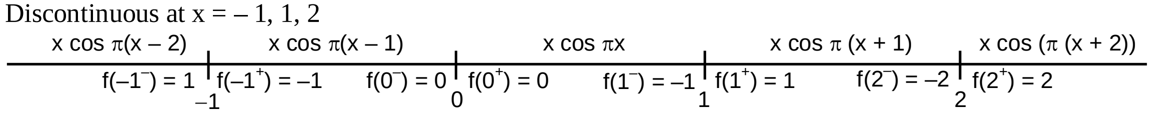
Q.41 Let $[x]$ be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ?

[A] $x = -1$ [B] $x = 0$

[C] $x = 2$ [D] $x = 1$

Sol. **A, C, D**

Discontinuous at x = – 1, 1, 2



* Q.42 If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle ?

[A] $2a$, 4, 1 [B] $2a$, 8, 1

[C] a, 4, 1 [D] a, 4, 2

Sol. **B, C, D**

Tangent to $\frac{x^2}{a^2} - \frac{y^2}{4^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - 16}$

Comparing with y = 2x + 1

m = 2

$\Rightarrow 4a^2 - 16 = 1$

$a^2 = \frac{17}{4}$

$a = \frac{\sqrt{17}}{2}$

Only 2a, 4, 1 are sides of a right-angled triangle

Q.43 Let $f : R \rightarrow (0, 1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1) ?

[A] $e^x - \int_0^x f(t) \sin t \, dt$

[B] $x^9 - f(x)$

[C] $f(x) + \int_0^{\pi/2} f(t) \sin t \, dt$

[D] $x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$

Sol. **B, D**

[A] no solution for $e^x = \int_0^x f(t) \sin t \, dt, x \in (0, 1)$

As LHS is > 1 and RHS is less than 1

[B] Let $h(x) = x^9 - f(x)$

$h(0) = - f(0) < 0$

$h(1) = 1 - f(1) > 0$

by IVT (As h(x) is cont. in [0, 1])

$h(x) = 0$ will be for some $x \in (0, 1)$

[C] both are +ve function so no solution

[D] $h(x) = x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$

$h(0) = -ve$

$h(1) = +ve$

so as h(x) is cont. in [0, 1]

so $h(x) = 0$ for some $x \in (0, 1)$

SECTION 2 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : + 3 If only the bubble corresponding to all the correct answer is darkened.
Zero Marks : 0 In all other cases.

* Q.44 The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

Sol. **6**

Let sides be a – d, a, a + d, (d > 0)

$\Rightarrow$ $a^2 + (a - d)^2 = (a + d)^2$

$\Rightarrow$ $a = 4d$

$\Rightarrow$ sides are 3d, 4d, 5d

As area is 24

$\Rightarrow$ $\frac{1}{2} \times 3d \times 4d = 24$

$\Rightarrow$ $d = 2$

$\Rightarrow$ sides are 6, 8, 10

$\Rightarrow$ smallest side is 6.

* Q.45 For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?

Sol. **2**

$x^2 + y^2 + 2x + 4y - p = 0$

$(x + 1)^2 + (y + 2)^2 = p + 5$

$\Rightarrow p + 5 = 4$ (when it touches the x-axis)

$p = -1$

and $p + 5 = 5$ (when circle passes through the origin)

$p = 0$

$\Rightarrow$ Two possible values of p

Q.46 For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Sol. **1**

Here, $D = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$

which gives $\alpha = -1$ or $+1$

For $\alpha = 1$, the equations become

$x + y + z = 1$

$x + y + z = -1$

and $x + y + z = 1$

which give no solution

For $\alpha = -1$, the equations become

$x - y + z = 1$

$-x + y - z = -1$

$x - y + z = 1$

which are all same and hence infinitely many solutions

Hence, $\alpha = -1$

$\Rightarrow 1 + \alpha + \alpha^2 = 1$

* Q.47 Words of length 10 are formed using the letters, A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated ; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

Sol. **5**

$x = 10!$

$y = {}^{10}C_1 \times {}^{9}C_8 \times \frac{10!}{2!}$

$\Rightarrow \quad \frac{y}{9x} = \frac{{}^{10}C_1 \times {}^{9}C_8}{9 \times 2!} = \frac{10 \times 9}{9 \times 2} = 5$

$\Rightarrow \quad 5$

Q.48 Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\pi/2} \left[f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t \, f(t) \right] dt$$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x \to 0} g(x) =$

Sol. **2**

$$g(x) = \int_x^{\pi/2} \left[f'(t)\operatorname{cosec} t - \cot(t)\operatorname{cosec}(t) f(t) \right] dt$$

$$\Rightarrow g(x) = f(t)\operatorname{cosec}(t)\Big|_x^{\pi/2} = 3 - \frac{f(x)}{\sin x}$$

$$\Rightarrow \lim_{x \to 0} g(x) = \lim_{x \to 0} \left(3 - \frac{f(x)}{\sin x}\right)$$

$$= 3 - f'(0) = 2$$

SECTION 3 (Maximum Marks: 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : + 3 If only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : – 1 In all other cases.

Answer Q.49, Q.50 and Q.51 by appropriately matching the information given in the three columns of the following table.

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

* Q.49 The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination ?

[A] (II) (iii) (R) [B] (IV) (iv) (S)
[C] (IV) (iii) (S) [D] (II) (iv) (R)

Sol. **D**

Tangent at $\left(\sqrt{3}, \frac{1}{2}\right)$ is $\sqrt{3}x + 2y = 4$

Since slope of tangent at $\left(\sqrt{3}, \frac{1}{2}\right)$ is –ve hence possible curve are (I), (II) only

⇒ equation of curve is $\frac{x^2}{4} + y^2 = 1$, i.e., $x^2 + a^2y^2 = a^2$

Hence equation of tangent is $y = mx + \sqrt{a^2m^2 + 1}$

And point of contact is $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$

* Q.50 If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only CORRECT combination ?

[A] (III) (i) (P) [B] (III) (ii) (Q)
[C] (II) (iv) (R) [D] (I) (ii) (Q)

Sol. **A**

Tangent at (8, 16) is $y = x + 8$

Slope of tangent is +ve hence possible curve will be $y^2 = 4ax$

⇒ equation of tangent is $my = m^2x + a$ and point of contact is

$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

* Q.51 For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1, 1), then which of the following options is the only CORRECT combination for obtaining its equation ?

[A] (II) (ii) (Q) [B] (III) (i) (P)
[C] (I) (i) (P) [D] (I) (ii) (Q)

Sol. **D**

For $a = \sqrt{2}$ and point (–1, 1) on the curve is $x^2 + y^2 = a^2$ equation of tangent is $y = mx + a\sqrt{m^2+1}$ and point of contact is $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$

Answer Q.52, Q.53 and Q.54 by appropriately matching the information given in the three columns of the following table.

Let $f(x) = x + \log_e x - x\log_e x$, $x \in (0, \infty)$.

- Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in (0, 1)
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0, 1)
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

Q.52 Which of the following options is the only CORRECT combination ?

[A] (IV) (i) (S) [B] (I) (ii) (R)

[C] (III) (iv) (P) [D] (II) (iii) (S)

Sol. **D**

$f(x) = x + \ell nx - x\ell n\, x$

$f'(x) = \frac{1}{x} - \ell nx$

$f''(x) = -\frac{1}{x^2} - \frac{1}{x} = -\frac{(1+x)}{x^2}$

$f'(1) \cdot f'(e) = \left(\frac{1}{e} - 1\right) < 0 \Rightarrow f'(x) = 0\ \forall$ some $x \in (1, e)$

$\lim_{x\to\infty}\left(\frac{1}{x} - \ell nx\right) = -\infty$

$f'(x)$ is decreasing for $x \in (0, \infty)$

Q.53 Which of the following options is the only CORRECT combination ?

[A] (III) (iii) (R) [B] (I) (i) (P)

[C] (IV) (iv) (S) [D] (II) (ii) (Q)

Sol. **D**

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} 1 - (x-1)(\ell nx - 1) = -\infty$

$f'(e) = \frac{1}{e} - 1 < 0$, $f''(x) < 0$

$\Rightarrow$ $f(x)$ is decreasing in (e, e^2)

Q.54 Which of the following options is the only **INCORRECT** combination ?

[A] (II) (iii) (P) [B] (II) (iv) (Q)

[C] (I) (iii) (P) [D] (III) (i) (R)

Sol. **D**

$f'(x)$ is decreasing and $f'(1) = 1$

$\Rightarrow$ $f'(x) = 0$ has no root in (0, 1)